Decomposition of Turbulence Forcing Field and Structural Response

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Theme

MEASURED cross-spectrum of a turbulence field usually shows some decay in the statistical correlation in addition to convention at a characteristic velocity. Under such a random excitation the computation of structural response statistics becomes much more tedious than that which would be the case if the turbulence were convected without decay, i.e., convected as a frozen-pattern. It is shown that a decaying turbulence can be decomposed into frozen-pattern components thus permitting a simpler way to calculate the structural response. The procedure so devised also provides a relationship whereby the measured input spectra can be incorporated. For illustration the theory is applied to an infinite beam which is backed on one side by a fluid-filled cavity and is exposed on the other side of the turbulence excitation. The effect of the freestream velocity is also taken into consideration.

Contents

When computing structural response to turbulent flow excitations the forcing field is often idealized as a frozen random pattern which is statistically homogeneous and is convected downstream at a known velocity. Restricting our discussion to only one spatial coordinate, this assumption implies that the forcing field has a Fourier-Stieltjes integral representation

$$p(x-U_c t) = \int_{-\infty}^{\infty} e^{i(\omega t - kx)} \mathrm{d}F(k) \tag{1}$$

in which the frequency ω and wave number k are related to the convection speed U_c as $\omega/k = U_c$. It is known from the random process theory that

$$E\{\mathrm{d}F(k_1)\,\mathrm{d}F^*(k_2)\} = S_p(k_1)\,\delta(k_1-k_2)\,\mathrm{d}k_1\mathrm{d}k_2 \tag{2}$$

where $E\{\ \}$ indicates an ensemble average, an asterisk denotes the complex conjugate, and $S_p(k)$ is the wave-number spectral density of the pressure field. Alternatively, the pressure field can also be characterized by a cross-spectral density in the frequency domain, $\Phi_p(\omega,\xi)$, referring to a fixed spatial coordinate,

$$\Phi_p(\xi,\omega) = (1/|U_c|)\,S_p(\omega/U_c)\,e^{-i(\omega/u_c)\xi} \tag{3}$$

where $\xi = x_1 - x_2$ is the spatial separation. Note that ξ appears only in the imaginary exponent in Eq. (3) when the pressure is truly a frozen pattern.

The motion of a structure responding to the excitation of a convected frozen pattern can be expressed in terms of a fundamental solution which is the response to a convected unit frozen sinusoid $\exp[i(\omega t - kx)]$. Let this fundamental solution be $H(x,k)\exp(i\omega t)$. In a certain sense, H is a frequency response function. Then the cross-spectral density of the structural response may be expressed in either of the following forms:

in the wave number domain

$$S_w(x_1,x_2;k) = H(x_1,k)\,H^*(x_2,k)\,S_p(k) \tag{4}$$

in the frequency domain

$$\Phi_w(x_1,x_2;\omega) = H(x_1,\omega/U_c)\,H^*(x_2,\omega/U_c)\,\Phi_p(0,\omega) \tag{5}$$

However, measured frequency cross-spectra often differ significantly from the ideal form, Eq. (3). For example, Corcos[1] has proposed the following general form for boundary-layer turbulence

$$\bar{\Phi}_p(\xi,\omega) = \bar{\Phi}_p(0,\omega)\,\psi(\xi)\,e^{(-i\omega\xi/U_c)} \tag{6}$$

where ψ is an even non-negative definite function of ξ, having an absolute maximum equal to one at $\xi=0$ and approaching zero at large absolute values of ξ. The ξ dependent function ψ is indicative of decay in the turbulence. When spatial decay is significant, the simple equation, (4) or (5) is no longer applicable.

It is proposed in this paper that a general turbulent pressure field can be constructed from superposition of infinitely many frozen-pattern components; i.e.

$$p(x,t) = \int_{-\infty}^{\infty} \hat{p}(x-ut)\,\mathrm{d}G(u) \tag{7}$$

Such a superposed pressure field has a theoretical frequency cross spectrum

$$\Phi_p(\xi,\omega) = \int_{-\infty}^{\infty} \frac{1}{|u|} e^{-i\omega\xi/u} S_p\left(\frac{\omega}{u},u\right)\mathrm{d}u \tag{8}$$

Then by equating the theoretical and experimental spectra, $\Phi_p(\xi,\omega)$ and $\bar{\Phi}_p(\xi,\omega)$, we find that

$$S_p\left(\frac{\omega}{u},u\right) = \frac{1}{2\pi}\left|\frac{\omega}{u}\right|\int_{-\infty}^{\infty} e^{i\omega\xi/u}\bar{\Phi}_p(\xi,\omega)\,\mathrm{d}\xi \tag{9}$$

In particular, for the Corcos form, Eq. (6),

$$S_p(\omega/u,u) = |\omega/u|\,\Psi(\omega/u - \omega/U_c)\,\bar{\Phi}_p(0,\omega) \tag{10}$$

where

$$\Psi(v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(\xi)\,e^{i\xi v}\,\mathrm{d}\xi$$

Responding to the excitation of the superposed pressure field, Eq. (7), the frequency cross-spectrum of the structural motion can be computed from

$$\Phi_w(x_1,x_2;\omega) = \bar{\Phi}_p(0,\omega)\int_{-\infty}^{\infty} H(x_1,k)\,H^*(x_2,k) \times \Psi(k-\omega/U_c)\,\mathrm{d}k \tag{11}$$

As seen from this equation, only single integration is required to compute response spectrum for a one-dimensional structure. Thus, application of this equation results in tremendous savings in computer time compared with the traditional response spectrum calculation which requires a double-integration for one-dimensional structures. Furthermore, the method developed here allows direct incorporation of the measured spectrum in the calculation.

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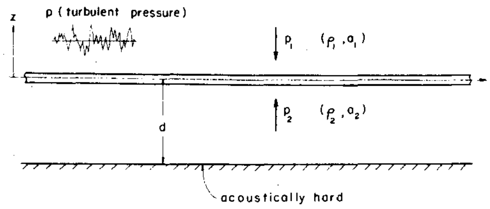

Fig. 1 An infinite beam under the excitation of boundary-layer turbulence.

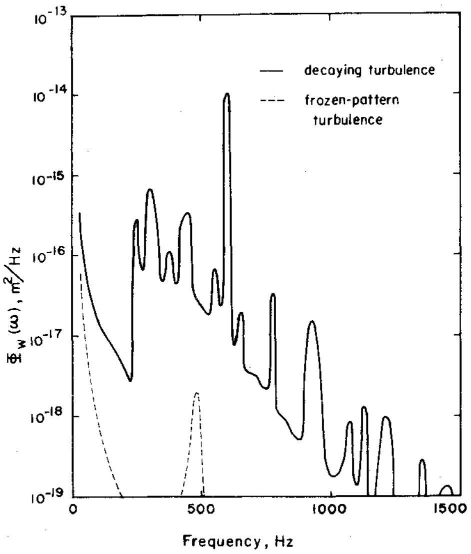

Fig. 2 Spectral density of structural response.

As an example, the theory is applied to an infinite beam shown in Fig. 1. The beam is backed on the lower side by a space of depth d which is filled with an initially quiescent fluid of density ρ_2 and sound speed a_2. On the upper side the beam is exposed to the excitation of a supersonic boundary-layer turbulent pressure p. The fluid on the upper side of the beam which carries the turbulence has a freestream velocity U_∞, density ρ_1 and sound speed a_1.

As the beam responds to the excitation its motion will generate additional pressures in the fluid media on the upper and lower sides. Denoting these generated pressures by p_1 and p_2, respectively, the governing equation of the beam motion is given by

$$EI\frac{\partial^4 w}{\partial x^4}+m\frac{\partial^2 w}{\partial t^2}=p+(p_1-p_2)_{z=0} \tag{12}$$

The governing equations and boundary conditions for p_1 and p_2 are

$$\left(\frac{\partial}{\partial t}+U_\infty\frac{\partial}{\partial x}\right)^2 p_1-a_1^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)p_1=0 \tag{13a}$$

$$\left(\frac{\partial p_1}{\partial z}\right)_{z=0}=\rho_1\left(\frac{\partial}{\partial t}+U_\infty\frac{\partial}{\partial x}\right)^2 w \tag{13b}$$

$$\frac{\partial^2 p_2}{\partial t^2}-a_2^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)p_2=0 \tag{14a}$$

$$\frac{\partial p_2}{\partial z}=0 \text{ at } z=-d,\quad \frac{\partial p_2}{\partial z}=\rho_2\ddot{w} \text{ at } z=0 \tag{14b}$$

In this case the frequency response H is found to be

$$H(x,k)=e^{(-ikx)}\{EIk^4-m\omega^2+i\rho_1 a_1\frac{k(\omega/k-U_\infty)^2}{[(\omega/k-U_\infty)^2-a_1^2]^{1/2}}+\rho_2\omega^2(\cot\gamma d)/\gamma\}^{-1} \tag{15}$$

where $\gamma^2=(\omega/a_2)^2-k^2$.

Figure 2 shows the computed results for the frequency spectrum (i.e., when $x_1=x_2$) of structural deflection using the following physical data:

properties of the beam

EI (bending rigidity) = 3.935×10^4 N-m^2

m (mass per unit length) = 9.746 kg/m

properties of the surrounding fluid media

$\rho_1=\rho_2=\rho$ (density) = 0.11015 kg/m^3

$a_1=a_2=a$ (speed of sound) = 261.6 m/sec

U_∞ (freestream velocity on upper side of beam) = 575.6 m/sec

U_c (convection velocity of the turbulence) = 0.75 U_∞

d (cavity depth) = 0.1178 m

properties of the turbulent pressure[2]

$$\psi(\xi)=\text{decay factor}=\exp\left(-\frac{|\xi|}{\alpha\delta}\right)$$

$$\bar{\Phi}_p(0,\omega)=\text{spectral density}=\frac{\delta}{2U_\infty}\sum_{n=1}^{4}A_n e^{-K_n(|\omega|\delta/U_\infty}$$

δ (boundary-layer thickness) = 0.279 m

and experimentally determined constants

$\alpha=3$

$A_1=4.4\times10^{-2}$	$K_1=5.78\times10^{-2}$
$A_2=7.5\times10^{-2}$	$K_2=2.43\times10^{-1}$
$A_3=-9.3\times10^{-2}$	$K_3=1.12$
$A_4=-2.5\times10^{-2}$	$K_4=11.57$

Also shown in Fig. 2 are the results obtained under the frozen-pattern assumption [i.e., by letting $\psi=1$ in Eq. (6)]. The comparison indicates that the neglect of spatial decay in the turbulence may lead to a grossly unconservative structural design. It may also lead to an underestimate of the noise level in the cavity.

In another paper,[3] evenly spaced supports are added to the infinite beam shown in Fig. 1. The structural model now resembles a typical aircraft fuselage construction, with multispan panels supported by stringers and frames. The supports gives rise to multiple reflections and the solution becomes much more complicated. Interested readers are referred to Ref. 3 for details.

References

[1]Corcos, G. M., "Resolution of Pressure in Turbulence," *Journal of the Acoustical Society of America*, Vol. 35, Feb. 1963, pp. 192-199.

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